That's a State Space Model too!

Michael D. Hunter

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Modern Modeling Methods (M^3) Storrs, CT; May 25, 2016

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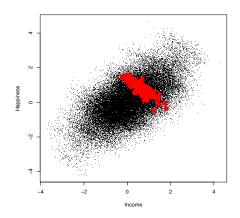
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Why? 00000 on References

Outline

- Introduction, Background, and Motivation
- State Space Models and Kalman Filters
- Examples
 - Factor Models
 - Standard Structural Equation Models
 - Latent Growth Models
 - Cross-Lagged Panel Models
 - Dual Change Score Models
 - Autoregressive Latent Trajectory Models
 - . . .
 - Discussion, Conclusions, and Future Work

Perspective



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Why? E

Where to go from here?

- Between-person models are valid, but (generally) only between people.
- Conclusions for individuals require repeated measurements for individuals.
- Model individuals and processes.
- Balance the Idiographic/Nomothetic trade-off

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Where to go from here?

- Between-person models are valid, but (generally) only between people.
- Conclusions for individuals require repeated measurements for individuals.
- Model individuals and processes.
- Balance the Idiographic/Nomothetic trade-off
- How do you model variability within people?

State Space, too

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State Space Model

Measurement

Structural Equation Measurement Model

$$oldsymbol{y}_{i} = \Lambda oldsymbol{\eta}_{i} + K oldsymbol{x}_{i} + oldsymbol{arepsilon}_{i}$$
 with $oldsymbol{arepsilon}_{i} \sim \mathcal{N}\left(oldsymbol{0}, \Theta
ight)$ (1)

State Space Measurement Model

$$oldsymbol{y}_i = \Lambda oldsymbol{\eta}_i + K oldsymbol{x}_i + oldsymbol{arepsilon}_i$$
 with $oldsymbol{arepsilon}_i \sim \mathcal{N}\left(oldsymbol{0}, \Theta
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State Space Model

Measurement

Structural Equation Measurement Model

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State Space Measurement Model

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Example 1 is done.

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Discussion

References

State Space Model

Measurement

Structural Equation Measurement Model

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ight)$ (2)

- Example 1 is done.
- OpenMx Notation

$$\boldsymbol{y}_i = C \boldsymbol{x}_i + D \boldsymbol{u}_i + \boldsymbol{r}_i$$
 with $\boldsymbol{r}_i \sim \mathcal{N}(\boldsymbol{0}, R)$ (3)
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State Space Model

Transition/Structural

Structural Equation Structural Model

$$\boldsymbol{\eta}_{i} = B \boldsymbol{\eta}_{i} + \Gamma \boldsymbol{x}_{i} + \boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}\left(\boldsymbol{0}, \Psi\right) \quad (4)$$

State Space Structural Model

$$\boldsymbol{\eta}_{i} = B \boldsymbol{\eta}_{i-1} + \Gamma \boldsymbol{x}_{i} + \boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}\left(\boldsymbol{0}, \Psi\right)$$
 (5)

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Transition/Structural

Structural Equation Structural Model

$$\boldsymbol{\eta}_{i} = B \boldsymbol{\eta}_{i} + \Gamma \boldsymbol{x}_{i} + \boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}\left(\boldsymbol{0}, \Psi\right) \quad (4)$$

State Space Structural Model

$$\boldsymbol{\eta}_{i} = B \boldsymbol{\eta}_{i-1} + \Gamma \boldsymbol{x}_{i} + \boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}\left(\boldsymbol{0}, \Psi\right) \qquad \textbf{(5)}$$

OpenMx Notation

$$oldsymbol{x}_i = Aoldsymbol{x}_{i-1} + Boldsymbol{u}_i + oldsymbol{q}_i$$
 with $oldsymbol{q}_i \sim \mathcal{N}\left(oldsymbol{0}, Q
ight)$ (6)

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State Space Model

OpenMx Notation



$$\boldsymbol{x}_{i} = A\boldsymbol{x}_{i-1} + B\boldsymbol{u}_{i} + \boldsymbol{q}_{i}$$
 with $\boldsymbol{q}_{i} \sim \mathcal{N}\left(\boldsymbol{0}, Q\right)$ (7)

Measurement

$$\boldsymbol{y}_i = C\boldsymbol{x}_i + D\boldsymbol{u}_i + \boldsymbol{r}_i \quad \text{with} \quad \boldsymbol{r}_i \sim \mathcal{N}\left(\boldsymbol{0}, R\right) \quad \textbf{(8)}$$

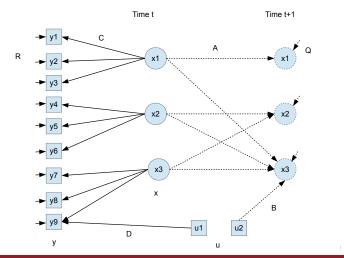
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State Space Model

Diagrams



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State Space Model

Available now in OpenMx

- Differential Equation in Discrete Time
- Implemented by me in OpenMx 2.0 Release
- Continuous Time is in OpenMx 2.1 Release

$$\frac{d}{dt}\boldsymbol{x}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}_i + \boldsymbol{q}(t)$$
(9)

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Kalman Filter Benefits

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- Designed for non-stationary time series
 - Cf. block-Toeplitz autocovariances (Molenaar, 1985)
 - Cf. lagged observed variables (Song & Zhang, 2014)
 - Cf. exact discrete model (Voelkle & Oud, 2013; Driver, Oud, & Voelkle, 2015)
- Gaussian noise: gives ML estimates
- Non-Gaussian noise: becomes least squares optimal
- Latent State Estimates are factor scores (Priestley & Subba Rao, 1975)
- Latent Covariace Estimates \Rightarrow Reliability (Hunter, In Preparation)

Kalman Filter

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Equations

Predict Step

$$\boldsymbol{x}_{t|t-1} = A\boldsymbol{x}_{t-1|t-1} + B\boldsymbol{u}_t \tag{10}$$

$$P_{t|t-1} = AP_{t-1|t-1}A^{\mathsf{T}} + Q \tag{11}$$

Update Step

$$\widehat{\boldsymbol{y}}_t = \widehat{Mean}(\boldsymbol{y}_t) = C\boldsymbol{x}_{t|t-1} + D\boldsymbol{u}_t$$
 (12)

$$\widetilde{\boldsymbol{y}}_t = \widehat{Residual}(\boldsymbol{y}_t) = \boldsymbol{y}_t - \widehat{\boldsymbol{y}}_t$$
 (13)

$$\widehat{S}_t = \widehat{Cov}(\boldsymbol{y}_t) = CP_{t|t-1}C^{\mathsf{T}} + R$$
(14)

$$K = P_{t|t-1}C^{\mathsf{T}}\widehat{S}_t^{-1} \tag{15}$$

$$\boldsymbol{x}_{t|t} = \boldsymbol{x}_{t|t-1} + K \ \widetilde{\boldsymbol{y}}_t$$
 (16)

$$P_{t|t} = P_{t|t-1} - KCP_{t|t-1} \tag{179 HSC}$$

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What about multiple individuals?



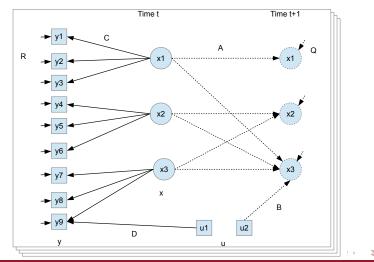
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The Factor Model?



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The Standard Structural Equation Model?



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Why?

SEM as a State Space Model Math ... go figure

The standard structural equation model is a factor model.

$$\boldsymbol{\eta}_{i} = B\boldsymbol{\eta}_{i} + \Gamma \boldsymbol{x}_{i} + \boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}\left(\boldsymbol{0}, \Psi\right)$$
 (18)

$$\boldsymbol{\eta}_{i} - B\boldsymbol{\eta}_{i} = (I - B)\boldsymbol{\eta}_{i} = \Gamma\boldsymbol{x}_{i} + \boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}(\mathbf{0}, \Psi) \quad (19)$$

$$\boldsymbol{\eta}_{i} = (I - B)^{-1}\Gamma\boldsymbol{x}_{i} + (I - B)^{-1}\boldsymbol{\zeta}_{i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}(\mathbf{0}, \Psi) \quad (20)$$

$$\boldsymbol{\eta}_{i} = \Gamma_{2}\boldsymbol{x}_{i} + \boldsymbol{\zeta}_{2,i} \quad \text{with} \quad \boldsymbol{\zeta}_{i} \sim \mathcal{N}\left(\mathbf{0}, (I - B)^{-1}\Psi(I - B)^{-\mathsf{T}}\right) \quad (21)$$

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The Latent Growth Model?



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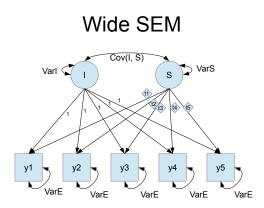
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Latent Growth Curve

Tucker (1958) & Rao (1958)



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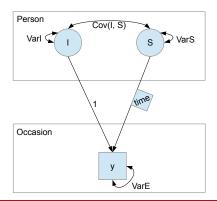
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Latent Growth Curve

Tucker (1958) & Rao (1958)

Relational SEM





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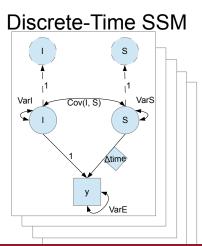
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Latent Growth Curve

Tucker (1958) & Rao (1958)



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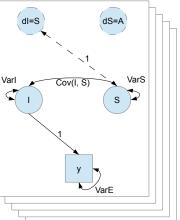
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Latent Growth Curve

Continuous-Time SSM





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The (Latent) Cross-Lagged Panel Model?



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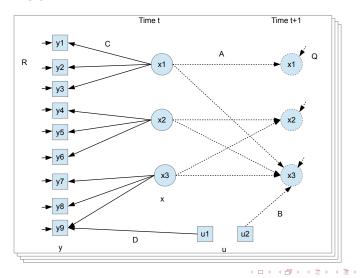


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Cross-Lagged Panel Model



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The Dual Change Score Model?



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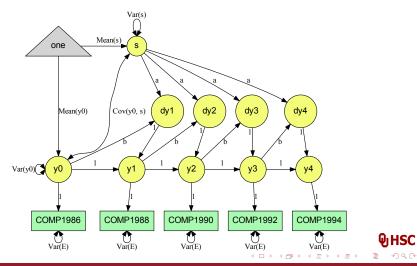
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Dual Change Score

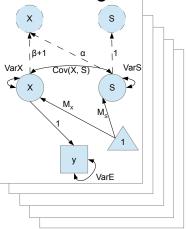


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Dual Change SSM



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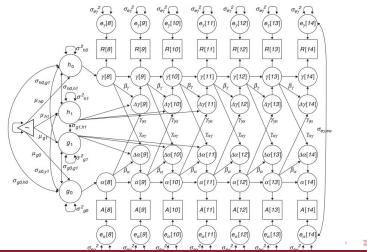
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Bivariate Dual Change Score

McArdle & Grimm (2010) "Five Steps"



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Examples 1-4

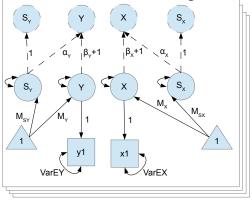
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The Autoregressive Latent Trajectory Model?



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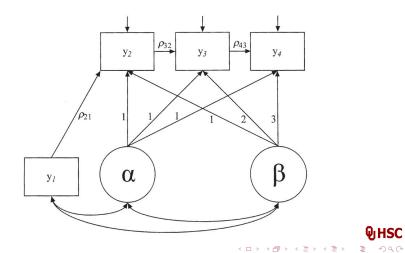
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Bollen & Curran (2004)



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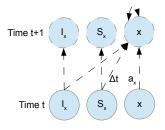
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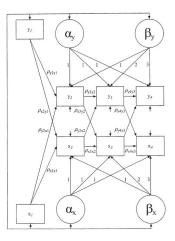
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Bivariate ALT

Bollen & Curran (2004)



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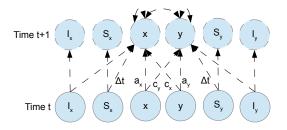
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Why? •0000

Why have I done this?

- 1. To see relationships among ModelsTM
- 2. To have a common way of expressing temporal relationships



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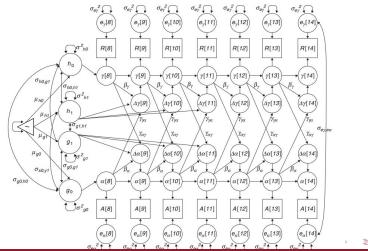
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Bivariate Dual Change Score

McArdle & Grimm (2010) "Five Steps"



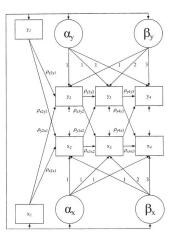
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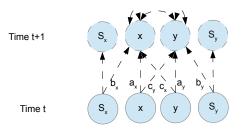
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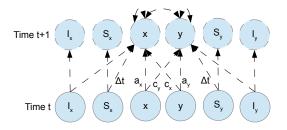
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Summary

- Between- and Within-person variabilities are distinct
- Within-person models are needed for within-person conclusions
- The discrete-time linear state space model encompasses many models of change in a single framework.
 - Factor Models
 - Standard Structural Equation Models
 - I atent Growth Models
 - Cross-Lagged Panel Models
 - Dual Change Score Models
 - Autoregressive Latent Trajectory Models

Common language is the foundation of communication.



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Future Work

- Emphasize continuous-time modeling for generalizable results.
- Integrate state space models with Item Factor Analysis.
- Integrate state space models with relational SEM.

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Why?

Acknowledgments

- OpenMx Core Development Team
- Sy-Miin Chow
- Come see "What's for dynr?"



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Thank You

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